11.2

Just U and I

Comparing Linear and Quadratic Functions

LEARNING GOALS

In this lesson, you will:

- Identify linear and quadratic functions from multiple representations.
- Compare graphs, tables, and equations for linear and quadratic functions.
- Analyze graphs of linear and quadratic functions.
- Determine if a function is linear or quadratic by analyzing the first and second differences.

KEY TERMS

- leading coefficient
- second differences

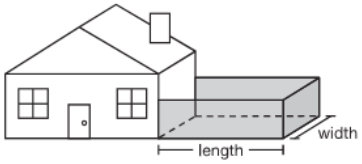
Where do we get the word "parabola"? Dictionaries tell us that the word comes from Greek, and is related to the word "parable."

In Greek, "para" means "beside" or "alongside." And "bole" means "throwing." The actual meaning of the word *parabola* comes from slicing a cone exactly parallel to its side. A parabolic shape is often used in satellite receivers to focus incoming signals to a central point. This type of shape is also used to send a beam of energy outward from a central point—like in a car's headlights.

Where else have you seen parabolas?

PROBLEM 1 Deciding on the Dimensions

Two dog owners have 16 yards of fencing to build a dog run beside their house. The dog owners want the run to be in the shape of a rectangle, and they want to use the side of their house as one side of the dog run. A rough sketch of what they have in mind is shown.



1. Complete the table to show different widths, lengths, and areas that can occur with sixteen yards of fencing.

Width	Length	Area
yards	yards	square yards
0		
2		
4		
6		
8		
10		
12		
14		
16		

2. Describe what happens to the length as the width of the dog run increases. Why do you think this happens?

3. Describe what happens to the area as the width of the dog run increases.

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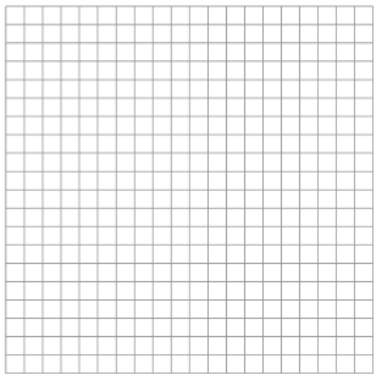
4. Describe what happens to the length and area as the width of the dog run decreases.

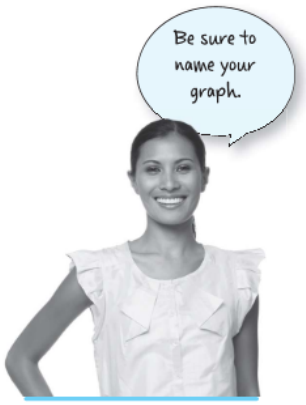
5. Describe what happens to the width and area as the length of the dog run increases.
Describe what happens to the width and area as the length of the dog run decreases.

6. Compare how the area changes as the width changes to how the area changes as the length changes.

7. Let $L(w)$ represent the length of the dog run as a function of the width. Create a graph to show this relationship. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable Quantity	Lower Bound	Upper Bound	Interval





8. Let $A(w)$ represent the area of the dog run as a function of the width. Create a graph to show this relationship. First, choose your bounds and intervals. Be sure to label your axes and name your graph.

Variable Quantity	Lower Bound	Upper Bound	Interval

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9. Let's compare and contrast the graphs of the two functions.
   $L(w)$: The length of the dog run as a function of the width.
   $A(w)$: The area of the dog run as a function of the width.

   a. Describe the type of function represented by each graph. Explain your reasoning.

   b. State the domain in terms of each function and the problem situation.

c. Determine the *y*-intercepts of each graph and interpret the meaning of each in terms of the problem situation.

d. Describe the rates of change for each graph.

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10. Determine the dimensions that provide the greatest area. Use the graphical representations to explain your reasoning.

PROBLEM 2 U and I

Tables A and B represent two different functions. One is a linear function, and one is a quadratic function.

Table A

x	A(x)
−2	7.5
−1	7.25
0	7
1	6.75
2	6.5

Table B

x	B(x)
−2	−15
−1	−7.25
0	0
1	6.75
2	13

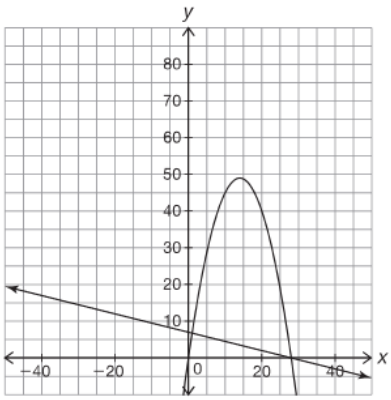
1. Which table do you think represents each type of function? Explain your reasoning.

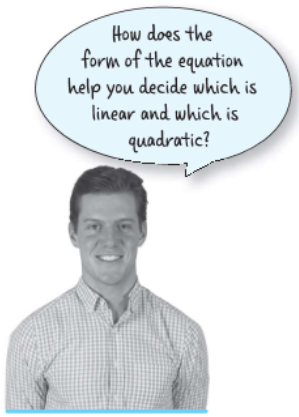
2. Calculate the first differences for each function. What patterns do you notice?

The graphs of the two functions are shown. The two equations that represent the linear and quadratic graphs are:

$$y = -\frac{1}{4}x + 7$$

$$y = -\frac{1}{4}x^2 + 7x$$





3. Identify the graph that represents Table A and the graph that represents Table B. Then rewrite each equation as the function $A(x)$ or $B(x)$ and label the graph appropriately. Was your prediction in Question 1 correct?

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4. Describe the rate of change for each graph. Explain your reasoning.

5. Determine the y-intercept of each function. Explain how you know.

The **leading coefficient** of a function is the numerical coefficient of the term with the greatest power. Recall that a power has two elements: the base and the exponent.

6. Identify the leading coefficient of each function. Then, describe how the sign of the leading coefficient affects the behavior of each graph.

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Let's explore the table of values one step further and analyze the *second differences*. **Second differences** are the differences between consecutive values of the first differences.

7. Calculate the second differences for each function. What do you notice?

PROBLEM 3 Second Differences

1. Analyze the form of each equation and determine if it is linear or quadratic. Then complete each table to calculate the first and second differences.

a. $y = 2x$ _____

x	y	First Differences	Second Differences
−3			
−2			
−1			
0			
1			
2			
3			

b. $y = 2x^2$ _____

x	y	First Differences	Second Differences
−3			
−2			
−1			
0			
1			
2			
3			

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c. $y = -x + 4$ _____

x	y	First Differences	Second Differences
−3			
−2			
−1			
0			
1			
2			
3			

d. $y = -x^2 + 4$ _____

x	y	First Differences	Second Differences
−3			
−2			
−1			
0			
1			
2			
3			

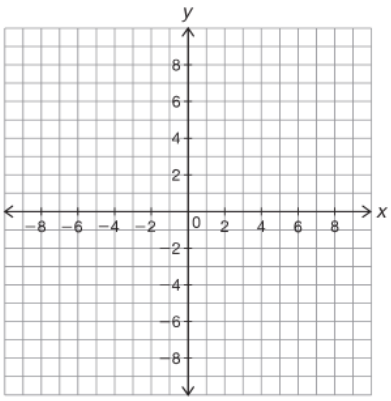
2. What do you notice about the first and second differences of the:

a. linear functions.

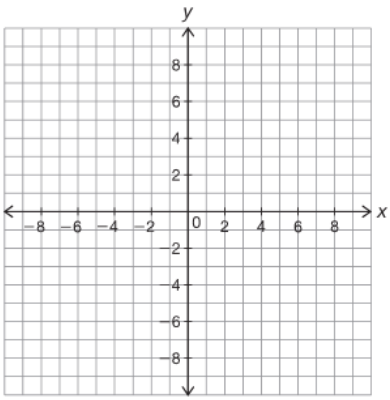
b. quadratic functions.

3. Sketch the graphs represented by the equations in Question 1.

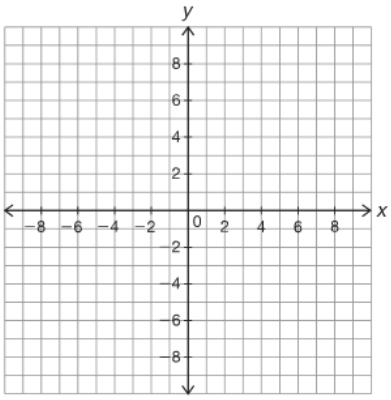
a. $y = 2x$



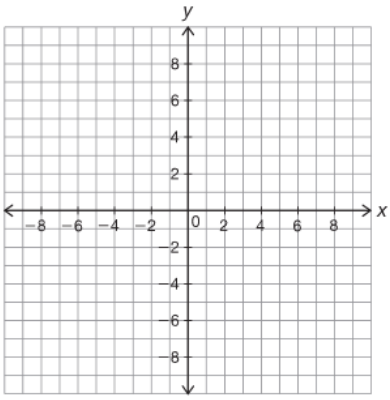
b. $y = 2x^2$



c. $y = -x + 4$



d. $y = -x^2 + 4$



4. Compare the signs of the first and second differences for each function and its graph.
   a. How do the signs of the first differences for a linear function relate to the graph either increasing or decreasing?

b. How do the signs of the first differences and the signs of the second differences for quadratic functions relate to the graph of the quadratic either increasing or decreasing or opening upward or downward?

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Talk the Talk

1. Describe how to determine when an equation represents a:
   a. linear function.

   b. quadratic function.

2. Describe how to determine when a table of values represents a:
   a. linear function.

   b. quadratic function.

3. Describe how the first and second differences describe the rate of change of a:
   a. linear function.

   b. quadratic function.

4. Describe how to determine when an equation represents a:
   a. linear function that increases.

   b. linear function that decreases.

   c. quadratic function that opens upward.

   d. quadratic function that opens downward.

5. Describe how to determine the *y*-intercept given any function.

Be prepared to share your solutions and methods.